

## David Begg's Full-employment Model

The model is given by the following equations:

$$Y = c(Y - T) + \dot{K} + \delta K + G \quad 0 < c < 1, \delta > 0 \quad (1)$$

$$Y = a_0 + a_1 K \quad a_0, a_1 > 0 \quad (2)$$

$$\frac{\dot{M}}{P} = b_1 Y - b_2 r \quad b_1, b_2 > 0 \quad (3)$$

$$r = a_1 + \pi \quad (4)$$

$$\frac{\dot{M}}{M} = \theta \quad (5)$$

where  $Y$  denotes aggregate real output,  $T$  real net tax revenue,  $K$  the real stock of capital,  $G$  real government expenditure,  $M$  the nominal money stock,  $P$  the price level,  $r$  the nominal interest rate, and  $\pi$  the rate of inflation. Equation (1) sets output equal to the sum of consumption ( $cY$ ), gross investment and government purchases. Gross investment is the sum of net investment,  $\dot{K}$ ,<sup>1</sup> and depreciation or replacement investment,  $\delta K$ . Equation (2) is a simple linear production function, with the role of labor suppressed on the presumption that there is full employment of labor. Equation (3) is a standard money-demand relationship; and (4) sets the nominal interest rate equal to the marginal product of capital plus the rate of inflation,  $\pi$ , which is equivalent to saying that the real interest rate equals the marginal product of capital.

We begin the analysis by deriving the dynamic equation for  $K$ , using (2) in (1):

$$\dot{K} = a_0(1 - c) + [a_1(1 - c) - \delta]K - G + cT$$

from which we can derive the stationary locus for  $K$  (setting  $\dot{K} = 0$ )

$$K^* = \frac{a_0(1 - c) - G + cT}{\delta - a_1(1 - c)}$$

This locus is vertical in a space with  $K$  on the  $x$ -axis and  $\frac{M}{P}$  on the  $y$ -axis: the steady-state value of the stock of capital does not depend on the level of real balances. Consider the situation if there is no government sector ( $G = T = 0$ ). In that case the steady-state value of the stock of capital is positive only if  $\delta - a_1(1 - c) > 0$  (since  $a_0(1 - c)$  is assuredly positive). We assume this to be the case. On this same assumption, the coefficient on  $K$  in the  $\dot{K}$  equation is negative. That says that if we 'step off' the stationary locus to the right, raising  $K$  at a given value of  $\frac{M}{P}$ , then  $\dot{K}$  will go negative, shifting us leftward back toward the locus. This is therefore a stable locus.

Now we turn our attention to  $\frac{M}{P}$ . Using (2) and (4) in (3) we get

$$\frac{\dot{M}}{P} = (b_1 a_0 - b_2 a_1) + b_1 a_1 K - b_2 \pi$$

For  $(\dot{M}/P)$  to equal zero, we need to have the price level rising at exactly the same rate as the money stock is expanding, i.e.,  $\pi = \theta$ . The stationary locus for real balances is therefore given by

$$\frac{M}{P} = (b_1 a_0 - b_2 a_1) + b_1 a_1 K - b_2 \theta$$

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<sup>1</sup>The dot notation here indicates the time-derivative of a variable, i.e.,  $\dot{K} \equiv dK/dt$ .

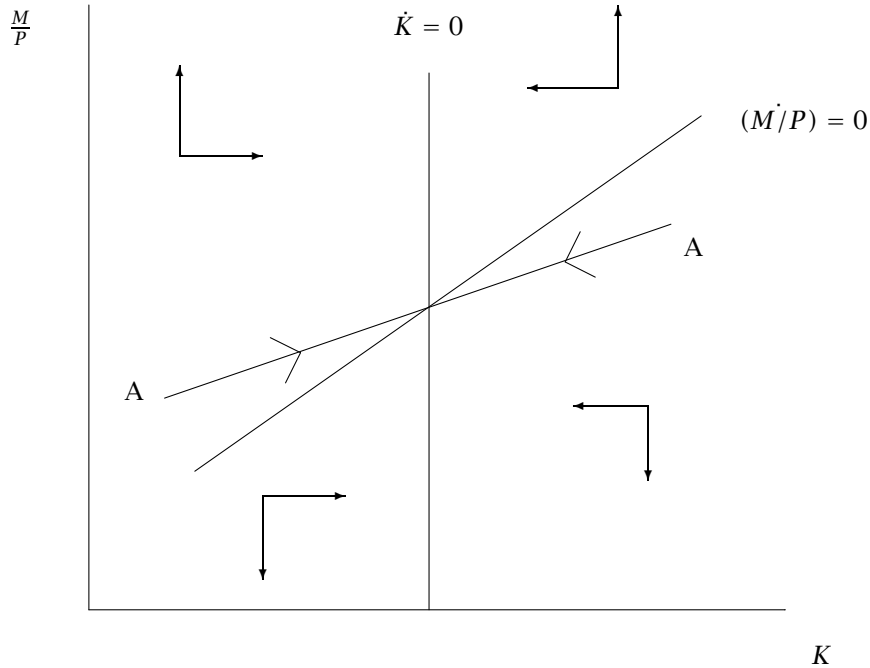


Figure 1: The Saddlepoint Property of the Begg Model

In the relevant space, this locus has the slope  $b_1 a_1 > 0$ . What about its stability? Let us first note that since the real money stock is the ratio of  $M$  to  $P$ , the proportional growth rate of real balances is the difference between the proportional growth of nominal money stock ( $\theta$ ) and the proportional growth rate of prices (i.e., inflation,  $\pi$ ). That is,

$$\frac{(M/P)}{(M/P)} = \theta - \pi$$

Since the *proportional* growth rate of real balances will always have the same sign as the simple *time-derivative* of real balances, we can work in terms of the former. From equation (4),  $\pi = r - a_1$  and from (3),  $r = \frac{b_1}{b_2} Y - \frac{1}{b_2} \frac{M}{P}$ . Using this information along with equation (2) we get

$$\frac{(M/P)}{(M/P)} = \theta + (a_1 - \frac{b_1}{b_2} a_0) - \frac{b_1}{b_2} a_1 K + \frac{1}{b_2} \frac{M}{P}$$

In this light, consider ‘stepping off’ the stationary locus for real balances in an upward direction (raising  $\frac{M}{P}$  at given  $K$ ). We start, on the locus, with  $(M/P) = 0$ , and raising  $\frac{M}{P}$  will then, via the positive coefficient  $1/b_2$ , generate a positive value of  $(M/P)$ . That is, the motion will be upward, away from the stationary locus. This locus is unstable. We can deduce that this model has the saddlepoint property. This is shown in Figure 1, where, given the arrows of motion, there exists a unique convergent path AA.

Within this model, what happens if there is an increase in government deficit spending (an increase in  $G$  without a corresponding increase in  $T$ )? Well, the stationary locus for real balances is unaffected: it stays where it is. But the stationary locus for capital stock is shifted. Working from the equation for  $K^*$  given above, we see that this locus moves to the left. That is, the economy will end up with a smaller stock of capital in the steady state. This occurs because the use of resources represented by the extra government spending means that less resources are available, out of any given income level,

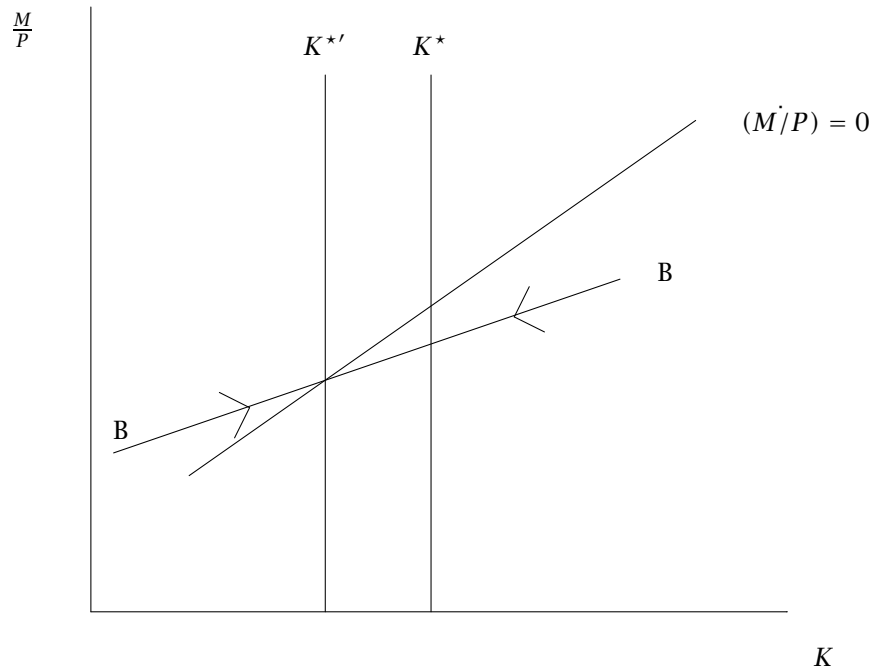


Figure 2: The Situation after an Increase in  $G$

to maintain the stock of capital. The new steady state will also show a lower level of real balances. This is because with lower  $K$ , the level of real output will also be lower. Hence the demand for real balances will be less: in equilibrium, the supply (stock) of real balances must also be smaller.

That is the comparison of the new steady state with the old. What about the dynamics of transition between the two states? Figure 2 shows the situation. The new convergent path is indicated as  $BB$ : it must lie beneath the old one. If the economy were to remain, for a while, at what *used to be* its steady state, the dynamic forces at play would tend to move it further away from the new steady state. What must happen, in order to initiate movement toward the new steady state? The economy must ‘jump’ onto  $BB$ . Since the capital stock is not a variable capable of jumping, and  $M$  is given at any point in time, it must be the price level that jumps up in order to reduce real balances and get us onto the new convergent path.

Clearly, for this model to work—that is, for the system to be able to converge after a disturbance to  $G$  or  $T$  (or to  $\theta$  for that matter)—the price level must be free to jump at a point in time. This is unrealistic. The price level is generally reckoned to be a somewhat ‘sticky’ variable. But the point of this exercise is to familiarize you with the mechanics of analysing this general class of models, starting with a relatively simple example. The next task will be to analyse a more interesting (short-run) macromodel, also with the saddlepoint property. Here the ‘jump variable’ will be one that is indeed capable of jumping, namely the foreign exchange rate.

*Exercise:* I have shown above the analysis of the effects of a change in deficit spending. To get a better sense of the model, please produce a similar analysis of (a) the effects of an increase in the rate of growth of money supply,  $\theta$ , and (b) the effects of a step increase in the economy’s capital stock (in general the capital stock is not capable of ‘jumping’, but one could imagine the economy receiving a gift of capital from abroad).